data of Table 1. Similarly, caution must be exercised when using $\epsilon_a = 1.00$ in temperature determinations from total radiometry and in particular with radiometry through an 8- to 14-μ filter.

Symbols.

$\alpha(\lambda)$	= absorptivity as a function of wavelength.
$(dT)_{\epsilon_a}$	= change in temperature at constant average emissivity.
$\epsilon(\lambda)$	= emissivity as a function of wavelength.
ϵ_a	= average emissivity, defined as $\int_2^{25}\epsilon(\lambda)W(\lambda, T)\,d\lambda/\int_2^{25}W(\lambda, T)\,d\lambda$.
$\epsilon_a^{\lambda_1-\lambda_2}$	= average emissivity for the wavelength range λ_1 to λ_2.
ϵ_a^T	= average emissivity at temperature T.
ϵ_a^0	= average emissivity extrapolated to 0°K from 325° to 450°K data.
λ	= wavelength.
$\rho(\lambda)$	= reflectivity as a function of wavelength.
R	= radiant energy.
σ	= Stefan's constant.
$s(\lambda)$	= fraction of scattered light as a function of wavelength.
$t(\lambda)$	= transmittance as a function of wavelength.
T	= absolute temperature.
$W(\lambda, T)$	= energy of a blackbody as a function of wavelength and temperature as defined by the Planck-Wien radiation law.

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Backscattering from an Undulating Surface with Applications to Radar Returns from the Moon

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Abstract. The backscattering properties of a smoothly undulated surface are discussed by means of a joint distribution of heights and surface slopes. It is shown that only those regions which are tilted so as to be normal to the incident radiation are effective in the backscattering. When this fact is properly accounted for, it is shown that the previously accepted scattering formulas have to be modified somewhat. It is also shown that the difference between the scattering properties of the two principal linear polarizations with respect to the mean surface must be exceedingly small for this kind of surface model.

…ntroduction. A large numer of theoretical …dies of backscattering from rough surfaces …e been published over the past few years. …st of these studies have been inspired by … stochastic nature of the reflection from the …osphere [*Feinstein*, 1954; *Ratcliffe*, 1956], by …ections from rough terrain or sea waves …*kmann and Spizzichino*, 1963], or by reflec…s from the moon and the planets [*Hargreaves*, …9; *Daniels*, 1961; *Hayre*, 1961; *Hagfors*, 1961; …*ter*, 1962; *Hughes*, 1962; *Fung*, 1964]. Nearly …these theories are based on the Huygens-…chhoff principle as formulated in the Helm…'s integral solution of the scalar wave equa…. The field over the surface is taken to be … sum of the incident and the reflected fields, … the local reflected field is set equal to the …dent field multiplied by the appropriate plane …e reflection coefficient which generally …nds on the local angle of incidence, unless, … often the case, the surface is taken to be a …ect reflector.

…he surface itself is considered to be gently …ulating and to be described sufficiently …rately by a joint distribution of two height …ations from a mean surface which is taken to …lane (ionosphere, earth terrain) or spherical …n, planets). The joint probability density … be defined for arbitrary separation between …wo points at which the two height deviations …easured.

… the present paper we shall make exactly …me type of assumptions as those outlined … to derive the scattering properties of a … surface. The main purpose of this paper is to point out that the previous work in this field has failed to take into account the statistical relationship between surface height deviation and local surface slopes, and that this relationship has a marked effect upon the angular scattering properties. *Fung* [1964] has recently also attempted such an approach, but has failed to arrive at the simple physical conclusions of this paper owing to, it is believed, a mathematical error.

In the particular case of a Gaussian distribution of surface height deviations, we are able to show explicitly what the relationship between height deviations and surface slopes means, and we are also able to prove an important result regarding the depolarization, or lack of depolarization, of waves scattered back at an arbitrary angle. This result states that the depolarization of the backscattered wave is the same for all angles of incidence as it is for normal incidence to a very good approximation. The same result has been stated in a more general form by *Beckmann and Spizzichino* [1963, see particularly p. 163], but their justification of the result invoked a rather artificial surface consisting of plane facets.

The results are applied to scattering from the lunar surface, and some modified angular power distributions are derived. In view of the theoretical results on polarization and the experimental depolarization results [*Evans and Pettengill*, 1963], it is pointed out that the search for better correlation functions for height deviations in order to match a larger portion of the angular spectrum of backscattering [*Muhle-*

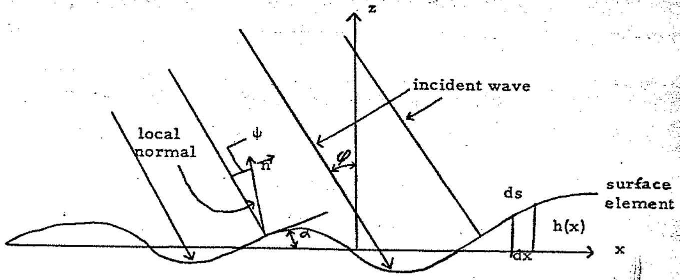

Fig. 1. Various quantities used in the analysis.

man, 1964; *Fung and Moore*, 1964] is probably physically meaningless. The reason is that very appreciable depolarization is indeed observed to take place in the tail of the echoes, and the quasi-specular model for the backscattering is, therefore, not a valid description for this part of the echo, whatever the form of the correlation function used.

Derivation of modified scattering formula. So as not to obscure the physical effects in too much mathematical detail, we consider only the one-dimensional surface shown in Figure 1. The mean surface is taken to be plane at $z = 0$, and the incident radiation is a plane wave of the form

$$E = E_0 \exp\left[-ik(x\cdot\sin\varphi - z\cdot\cos\varphi)\right] \qquad (1)$$

We now use the Helmholtz-Kirchhoff diffraction formula in the special form applicable for backscattering:

$$E_R = \frac{E_0 k}{2\pi i R} \exp(-ikR) \int ds\, Q(\psi) \cos\psi$$

$$\cdot \exp\left[-2ik(x \sin\varphi - h \cos\varphi)\right] \qquad (2)$$

Here R is the distance from the origin to the receiver, $Q(\psi)$ is the local plane wave reflection coefficient, ψ is the local angle of incidence, and h is the surface deviation at the point x. (In a strict sense we should use cylindrical waves rather than spherical waves in the present model. This distinction is, however, unimportant for the main arguments.)

The surface element ds can be expressed as

$$ds = dx/\cos\alpha \qquad (3)$$

We furthermore note that the local angle of incidence can be expressed in terms of the surface slope $t = \tan\alpha = dh/dx$ as follows:

$$\cos\psi = \cos(\varphi - \alpha)$$

$$= (\cos\varphi + t \sin\varphi)/\sqrt{(1 + t^2)}$$

$$\sin\psi = \sin(\varphi - \alpha)$$

$$= (\sin\varphi - t\cos\varphi)/\sqrt{(1 + t^2)}$$

Substituting into (2) and replacing the consta[nt] term in front of the integral sign by C, we obta[in]

$$E_R = C \int dx\, Q(t)(\cos\varphi + t\sin\varphi)$$

$$\cdot \exp(-2ik(x\sin\varphi - h\cos\varphi)$$

Most previous work has right from the outs[et] replaced ds in (3) by dx on the assumption th[at] the surface slopes are gentle, and on the sam[e] assumption neglected the term involving t[he] slope in $\cos\psi$ [e.g., *Hughes*, 1962; *Winter*, 196[2]]. In (5) we consider the reflection coefficient Q to be a function of x, since both t and h a[re] functions of position. The mean square field [at] the receiver takes the form:

$$\langle |E_R|^2\rangle_{av} = |C|^2 \iint dx\, dx' \langle Q(t)Q^*(t')$$

$$\cdot(\cos\varphi + t\sin\varphi)(\cos\varphi + t'\sin\varphi)$$

$$\cdot\exp\left[2ik\cos\varphi(h - h')\right]\rangle_{av}$$

$$\cdot\exp\left[-2ik\sin\varphi(x - x')\right]$$

By the following change in variables, $\Delta x = x -$ [...] and $\Delta h = h - h'$, we obtain

$$\langle |E_R|^2\rangle_{av} = |C|^2 \int dx \int d(\Delta x)\langle Q(t)$$

$$\cdot Q^*(t')(\cos\varphi + t\sin\varphi)(\cos\varphi + t'\sin\varphi)$$

$$\cdot\exp(2ik\cos\varphi\Delta h)\rangle_{av}\exp(-2ik\Delta x\sin\varphi)$$

[to] take both reflection coefficient variations and [th]e other local effects properly into account, we [ne]ed to know the joint probability density of [th]e height difference Δh over a surface interval and the slopes t and t' at the end points of [th]is interval. If we are permitted to assume a [Ga]ussian height deviation density function, [all]ow the rms height deviation to be unity, and [ass]ume a correlation $\rho(\Delta x)$ between the height [de]viations at the end points of the surface [int]erval, we can write the probability density [ex]plicitly in the form [*Rice*, 1944 and 1945]:

$$[p(\Delta]h, t, t') = (2\pi)^{-3/2}\,|M|^{-1/2}$$

$$\exp\left\{-\frac{1}{2|M|}\left(M_{11}\Delta h^2 + M_{22}(t^2 + t'^2)\right.\right.$$

$$\left.\left. 2M_{12}\Delta h(t - t') + 2M_{23}tt'\right)\right\} \qquad (8)$$

[wh]ere the matrix M is:

$$= \begin{Bmatrix} 2(1-\rho) & -\dfrac{\partial\rho}{\partial\Delta x} & \dfrac{\partial\rho}{\partial\Delta x} \\ -\dfrac{\partial\rho}{\partial\Delta x} & -\left.\dfrac{\partial^2\rho}{\partial\Delta x^2}\right|_{\Delta x=0} & -\dfrac{\partial^2\rho}{\partial\Delta x^2} \\ \dfrac{\partial\rho}{\partial\Delta x} & -\dfrac{\partial^2\rho}{\partial\Delta x^2} & -\left.\dfrac{\partial^2\rho}{\partial\Delta x^2}\right|_{\Delta x=0} \end{Bmatrix} \qquad (9)$$

[Th]e double-indexed quantities in (8) are the [co]factors of this matrix, and $|M|$ denotes the [de]terminant of the matrix.

Because the reflection coefficients Q are in [gen]eral fairly complicated functions of t, it will [usu]ally be quite difficult to evaluate the mean [...] (7) by means of the probability density [(e]quation 8). We therefore put

$$[F(t], t') = Q(t)Q^*(t')(\cos\varphi + t\sin\varphi)$$

$$\cdot(\cos\varphi + t'\sin\varphi) \qquad (10)$$

[an]d expand this in a power series about a slope [t]o be determined:

$$[F(t], t') = F(\bar t, \bar t) + \frac{\partial F}{\partial t}(t - \bar t)$$

$$+ \frac{\partial F}{\partial t'}(t' - \bar t) + \frac{\partial^2 F}{\partial t^2}\tfrac{1}{2}(t - \bar t)^2$$

$$+ \frac{\partial^2 F}{\partial t\partial t'}(t - \bar t)(t' - \bar t)$$

$$+ \frac{\partial^2 F}{\partial t'^2}\tfrac{1}{2}(t' - \bar t)^2 + \cdots \qquad (11)$$

It is obviously an advantage to expand about a $\bar t$ such that the first-order terms in the expansion (equation 11) vanish after integration over Δx in (7). Carrying out the integration,

$$\int d(\Delta x)\exp(-2ik\Delta x\sin\varphi)$$

$$\cdot\iiint dt\,dt'\,d(\Delta h)p(\Delta h, t, t')t$$

$$\cdot\exp(2ik\Delta h\cos\varphi)$$

under the assumption that the phase modulation is deep, i.e., $2kh_m > 1$, where h_m is the rms height deviation, and properly normalizing, we obtain for the value $\bar t$

$$\bar t = \tan\varphi \qquad (12)$$

The physical significance of this result is clear. Only those regions which are tilted so as to be normal to the incident wave are effective in the backscattering. We note in particular that in the zero-order term only the reflection coefficient for normal incidence enters, and there is thus no difference in the scattering properties of the two principal linear polarizations with respect to the mean surface in this approximation.

The zero-order approximation for the mean square electric field at the receiver becomes:

$$\langle |E_R|^2\rangle = \frac{E_0^2}{R^2\lambda^2\cos^2\varphi}|Q_0|^2$$

$$\cdot\int dx\int d(\Delta x)\exp\{-(2kh_m\cos\varphi)^2$$

$$\cdot[1 - \rho(\Delta x)]\}\exp(-2ik\Delta x\sin\varphi) \qquad (13)$$

where Q_0 is the reflection coefficient at normal incidence, i.e.,

$$|Q_0| = \left|\frac{\sqrt{\epsilon} - 1}{\sqrt{\epsilon} + 1}\right| \qquad (14)$$

The above result is identical to the one obtained by *Hughes* [1962] except for the factor $\cos^{-2}\varphi$ in front of the integral sign in (13) instead of $\cos^2\varphi$ obtained by Hughes.

The result of (13) can be extended to the two-dimensional case quite easily if the statistical properties of the surface are isotropic and the mean surface is plane:

$$\langle |E_R|^2 \rangle_{av} = \frac{E_0^2}{R^2 \lambda^2 \cos^2 \varphi} |Q_0|^2$$

$$\cdot \iint dx\, dy \int d(\Delta r) \Delta r$$

$$\cdot \exp\left[-(2kh_m \cos\varphi)^2 (1 - \rho(\Delta r))\right]$$

$$\cdot J_0(2k\Delta r \sin\varphi) \qquad (15)$$

where $J_0(2k\ \Delta r \sin\varphi)$ is the zero-order Bessel function.

If the reflection coefficient is independent of angle of incidence, (13) and (15) exactly account for the effects of variable local surface slopes. The results given here are not quite in agreement with those of *Fung* [1964], although he properly included surface slopes in his initial formula for the scattered power (see his equation 8). The discrepancy is thought to stem from an erroneous sign in his equation 13 resulting in a loss of the terms which are linear in the slope in his equation 16. Our result (equation 15) appears to be in agreement with some of *Beckmann and Spizzichino*'s [1963, see particularly section 5.3] results. Beckmann avoids the complication of the statistics of surface slopes by assuming the reflection coefficient to be constant and by carrying out a partial integration with respect to x in our (5), thus eliminating the term containing t.

Next we allow $Q(t)$ to vary with the local angle of incidence and carry the expansion to second order in $(t - \bar{t})$. The first-order terms drop out when we take the weighted means of the terms $(t - \bar{t})$ by definition. It also evolves that

$$\overline{(t - \bar{t})(t' - \bar{t})} = 0 \qquad (16)$$

$$\delta\langle |E_R|^2 \rangle_{av} = \pm \frac{2E_0^2 |Q_0|^2}{R^2 \lambda^2 \sqrt{\epsilon}} \cos^2\varphi\, h_m^2 \int dx \int d(\Delta x) \left(\frac{\partial^2 \rho}{\partial \Delta x^2} - \frac{\partial^2 \rho}{\partial \Delta x^2}\bigg|_{\Delta x = 0} \right)$$

$$\cdot \exp\{-(2kh_m \cos\varphi)^2 (1 - \rho(\Delta x))\} \exp(-2ik\Delta x \sin\varphi) \qquad (24)$$

so that the only terms of second order are the two of the form:

$$\frac{1}{2} \frac{\partial^2 F}{\partial t^2}\bigg|_{t=\bar{t}} \overline{(t - \bar{t})^2} \qquad (17)$$

To evaluate this, we first have to write the expression for the reflection coefficient and evaluate the second derivation at $t = \tan\varphi$. When the electric field is *in* the local plane of incidence, we have

$$Q_\parallel = \frac{\epsilon \cos\psi - \sqrt{\epsilon - \sin^2\psi}}{\epsilon \cos\psi + \sqrt{\epsilon - \sin^2\psi}} \qquad (18)$$

With this expression for Q we obtain

$$\frac{\partial^2 F}{\partial t^2}\bigg|_{t=\bar{t}} = -\frac{2}{\sqrt{\epsilon}} |Q_0|^2 \cos^4\varphi \qquad (19)$$

When the electric field is perpendicular to the local plane of incidence, the reflection coefficient is

$$Q_\perp = \frac{\cos\psi - \sqrt{\epsilon - \sin^2\psi}}{\cos\psi + \sqrt{\epsilon - \sin^2\psi}} \qquad (20)$$

With this expression for Q we obtain

$$\frac{\partial^2 F}{\partial t^2}\bigg|_{t=\bar{t}} = +\frac{2}{\sqrt{\epsilon}} |Q_0|^2 \cos^4\varphi \qquad (21)$$

i.e., the same as for the other polarization with opposite sign.

The other factor in (17) can be evaluated in the same way as we evaluated $\bar{t}$. We obtain

$$\overline{(t - \bar{t})^2} = -\frac{h_m^2}{I} \int d(\Delta x) \left(\frac{\partial^2 \rho}{\partial \Delta x^2} - \frac{\partial^2 \rho}{\partial \Delta x^2}\bigg|_{\Delta x = 0} \right)$$

$$\cdot \exp\{-(2kh_m \cos\varphi)^2 (1 - \rho(\Delta x))\}$$

$$\cdot \exp(-2ik\Delta x \sin\varphi) \qquad (22)$$

where

$$I = \int d(\Delta x) \exp\{-(2kh_m \cos\varphi)^2 (1 - \rho(\Delta x))\}$$

$$\cdot \exp(-2ik\Delta x \sin\varphi) \qquad (23)$$

The correction term which must be added to (13) to include second-order effects is therefore of the form

The positive sign is to be used when the electric field is perpendicular to the plane of incidence and the negative sign for the other polarization. Obviously the correction term is of the order $\kappa(h_m/l)^2$, where l is a scale length, and κ is a factor which is certainly much less than unity. The correction term (equation 24) is therefore normally exceedingly small.

It might seem surprising at first that the two polarizations contribute with different signs in the expression (equation 24). This, however, is due to the simplified model used. For the one-dimensional surface there *is* a difference between the two polarizations even at normal incidence. When going to a two-dimensional surface the correction term will have to be a weighted sum of the two different terms in (24). At normal incidence the two weight-factors must be equal, because any linear polarization is just as likely to be *in* the local plane of incidence as *across* this plane.

Let us only conclude here that (13) and (15) must be regarded as extremely good approximations to the backscattered power.

Application to lunar echoes. We shall give the modified results for the power versus angle of incidence in two particular cases; namely, for a Gaussian and for an exponential autocorrelation function of height deviations. In either case the height deviations are Gaussian and the phase modulation is taken to be deep, i.e., $kh_m \gg 1$. For the Gaussian case

$$\langle h(x) \cdot h(x + \Delta x) \rangle_{av}$$

$$= h_m^2 \rho(\Delta x) = h_m^2 \exp(-(\Delta x)^2/l^2) \qquad (25)$$

The power as a function of angle of incidence φ becomes

$$P(\varphi) = \cos^{-4}\varphi \exp(-l^2 \tan^2\varphi / 4h_m^2) \qquad (26)$$

For this model, the factor which must be used to multiply the smooth sphere scattering cross section to obtain the actual cross section (gain factor) is:

$$g_g = 1 + 2(h_m/l)^2 + \cdots \qquad (27)$$

The exponential case cannot strictly be dealt with in this formalism because the second derivative of the correlation coefficient ρ is infinite at the origin. However, we shall here assume that the exponential autocorrelation function

$$\rho(\Delta x) = \exp(-|\Delta x|/l) \qquad (28)$$

is only an approximation to a more complicated function which does indeed have finite derivatives at the origin. If we are justified in this argumentation, we obtain for the power versus angle of incidence

$$P(\varphi) = \left\{ \cos^4\varphi + \sin^2\varphi \frac{1}{4(kh_m)^2} \left(\frac{l}{h_m}\right)^2 \right\}^{-(3/2)} \qquad (29)$$

The gain factor in this case becomes

$$g_e = 1 + 2(h_m k)(h_m/l) + \cdots \qquad (30)$$

on the assumption that $(h_m k)(h_m/l) < 1$.

We shall not embark on a detailed discussion of lunar results here, but only point out that the analysis of lunar return with the formulas presented above would lead to slightly smaller rms slopes than have previously been deduced.

Another and possibly more important conclusion regarding moon echoes can be drawn from the following observation. *Evans and Pettengill* [1963], in their experimental work, found a very appreciable depolarization to take place toward the tail of the echo. In view of the results found above that would seem to indicate that only negligible depolarization should take place, it is suggested that an entirely different type of reflection mechanism should be sought to describe the tail of the echo. Any attempt to obtain the power versus range function much beyond the initial return from the smooth undulating surface model by the use of fancy autocorrelation functions should, therefore, not be taken too seriously.

Conclusions. In this paper we have shown that fairly important corrections result in the backscattering properties of an undulating surface when we take into account the statistical relationship of surface slopes and height deviations. We have also shown that on the model described the amount of depolarization observed should be a second-order effect in the slope parameter h_m/l.

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A Model for Computing Infrared Transmission through Atmospheric Water Vapor and Carbon Dioxide

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Abstract. Analytical expressions are developed for the direct computation of infrared transmission as a function of the amount, pressure, and temperature of the absorber. The parameters that are necessary for the application of the expressions are tabulated over the wave number range between 25 and 2150 cm^{-1} (400 to 4.65 microns in wavelength) in intervals of 25 cm^{-1}. Sample computations of the transmission described by the model have been shown to be comparable to recently published data. In its present form the model provides a practical tool for the numerical treatment of many radiation problems.

ntroduction. The distribution of atmoseric patterns of absorption and emission of g-wave radiation varies with evolving metelogical conditions and identifies sources and ks of energy. With the advent of infrared asurements from satellites, greater urgency s been placed on the adequate description, in e spectral detail, of the concurrent radiation d between satellite altitudes and the surface the earth. In lieu of networks for obtaining tine tropospheric radiation measurements, a venient numerical method could provide a ans for establishing relationships between ined data and significant atmospheric propies.

n any model of infrared radiative transfer paramount problem is that of properly specng the transmission through different spectral ons. If the model is oversimplified, significant iative properties may be overlooked or dised; if the model is too complex, it will not practicable for routine application to sizable ntities of data. In general, the minimum plexity demanded of a model is related to desired spectral resolution, and the signifie of any computation depends on the reliaty of the basic meteorological data.

or this investigation a transmission repretation was considered satisfactory if it met following requirements: (1) It should be oducible and practicable for objective apion without excessive empiricism or data ngs. (2) It should respond adequately to the able pressure, temperature, and absorber ditions of the troposphere and lower stratosphere. (3) It should be capable of representing the transmission over wave number intervals of about 50 cm^{-1} or less. (4) It should be compatible with available experimental data, and flexible enough to permit a simple adaptation to new data that become available.

Since many methods for describing transmission have been used, it is likely that different opinions exist regarding the relative merits of applied techniques. It is not likely that most models would completely satisfy all of the requirements listed above. The purpose of this report is to present a practical model for making direct computations of transmission for a variety of applications.

Empirical representation. An empirical representation of the transmission must be bounded by unity and zero. As these limits are approached, uncertainty arises when absorber amounts and pressures are encountered for which transmission data do not exist. In general, extensive testing is necessary to ensure that the transmission functions behave in a proper physical sense for all variations in the conditions to which they may be applied. One type of difficulty may arise in the spectral region of transition from one transmission form to another [cf. *Wark et al.*, 1964]. Also, within a given spectral interval, there are physical constraints to the derivatives of the transmission functions. For example, as a given layer is extended from a fixed level in the atmosphere toward lower pressure and temperature, even with only a very slight increase in absorber amount, the transmission should decrease. In contrast, the